

Figures of Merit and Optimal Design: An ECM Quadcopter Drone Lift Motor

ABSTRACT

It is common practice, in the selection of motors for a given application, to use figures of merit and datasheet parameters to compare one motor against another. Caution is warranted in such comparison to ensure that the definitions and assumptions used by each manufacturer are sufficiently consistent to make those comparisons valid.

A larger problem, perhaps, is that the numbers needed to establish that one motor is a better choice than another, in a given application, may not be readily apparent on the data sheet.

This note will support this claim by developing the optimal design of a drone lift motor, with respect to flight time, and showing that the terms involved differ from typical figures of merit. The resulting motor shows significant performance advantage, relative to a typical off-the-shelf option. This illustrates the advantage of ECM's ability to optimize motors to essentially arbitrary criteria.

This note begins with a brief discussion of some of the figures of merit and key parameters encountered on motor datasheets. We then consider the performance of a drone lift motor suitable for quadcopter drones and develop a flight-time optimal design for that application.

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1 Introduction

ECM builds and designs axial-flux motors with printed circuit board stators via an optimization process driven by constraints and optimization criteria that come from customer end-use requirements.

As such, there is no common recipe or canonical ECM motor that can be used to understand what the typical performance of an ECM motor looks like. Indeed, optimal designs tend to enhance (or diminish) the quantities being optimized at the expense of all other features of the design.

In this context of *design* rather than *selection*, the criteria of excellence for a given motor application are uniquely important. At ECM, we generally decline to optimize a motor for a figure of merit that is not directly related to the performance of the motor in an application. Usually, an ECM motor optimized for a specific

criterion, such as K_t , in the absence of constraints, is a poor design even if it compares favorably to another motor on the basis of that specific criterion.

2 Motor Parameters and Figures of Merit

2.1 K_t and K_v

Motor constants K_t and K_v are often involved in the context of motor selection. In the simplest sense, K_t is the relationship between motor current, I , and torque, τ , such that

$$\tau = K_t I \quad (1)$$

and K_v is the dual relationship between motor speed, ω , and voltage, V , i.e.,

$$V = K_v \omega. \quad (2)$$

Note that the speed and torque are through and across variables, as are current and voltage.

If the motor lacks losses and is linear, the equations above are a complete description. Multiplying the equations together yields

$$K_t I V = K_v \omega \tau. \quad (3)$$

Noting that mechanical power $P_m = \omega \tau$ and that electrical power is $P_e = I V$, and that there is no storage or loss modeled, the apparent conclusion is that $K_t = K_v$.

At this point, a dimensional analysis is in order. K_t has units of N m A^{-1} and K_v has units of V s rad^{-1} . The base units of N are kg m s^{-2} , and the base units of V are $\text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$. The dimensions are identical.

The numbers manufacturers use for K_t and K_v sometimes include losses or other phenomena that make the values different, or different units are used in the respective definitions. Currents and voltages are three-phase, so delta vs wye, RMS vs amplitude, and other conventions may come into play.

Regardless, one cannot increase K_t and reduce K_v , as they are fundamentally the same quantity.

In motor selection, K_v can be used to establish a minimum source voltage for the inverter required to drive the motor at a particular speed. That is, a larger K_v requires a higher voltage for a given speed.

Similarly, K_t can be helpful in terms of limits on current. A motor with a larger K_t requires less current to produce a certain torque. Whether a difference in current is meaningful depends on the resistances involved in the system.

ECM typically handles K_v and K_t related constraints by calculating a full motor operating point, including temperature, and constraining the currents and voltages directly at those operating points to suit the power electronics and supply limits. This is a more complete analysis that incorporates nonlinear effects, motor resistance, and otherwise reflects the actual operating condition.

2.2 The motor constant, K_m

The “motor constant” K_m proposes, in some ways, to address what K_t does not – the efficiency of torque production.

A common definition of K_m is as follows

$$K_m = \frac{\tau}{\sqrt{P}} \quad (4)$$

where τ is torque and P is *resistive* power loss. If R_t is the lumped resistance on the electrical terminal side of a machine, the resistive power loss is

$$P = I^2 R_t \quad (5)$$

and the torque developed is

$$\tau = K_t I. \quad (6)$$

Thus,

$$K_m = \frac{K_t}{\sqrt{R_t}}. \quad (7)$$

This latter definition also appears on Wikipedia, and is advocated by Moog components for use in assessing competitors' datasheets. The basic assumption – that R_t captures the loss in the motor – is *dependent on a lumped motor model* and distribution of loss that is largely accurate for motors with core material. There will of course be other losses in that motor. Note that square root in the denominator makes this quantity independent of operating current I .

There are some who define K_m as the ratio of torque to electric input power, note that this will make the resulting quantity dependent on I , and hence, a specific operating point.

ECM finds K_m to be useful in a limited sense. The losses under nominal conditions in a conventional BLDC may well be captured sufficiently by winding resistance to allow ballpark comparisons between similar motors. In an ECM PCB stator motor, the resistance R_s can be made very small, and very high currents may be accommodated due to the thermal structure of the stator. Further, the applications where K_m seems most useful are also applications in which high speeds and high peak torques are occasionally important. Those operating regimes will expose loss mechanisms and limitations, such as nonlinear torque as a function of current [1], that are not well captured by comparing K_m values based on parameters at nominal conditions.

In ECM's optimal design framework, the preference is to minimize the losses of a high-torque operating point representing the motor's actual intended operating condition. In effect, this is closely related to K_m , but takes nonlinearity and thermal effects into account.

2.3 (specific) torque and power (density)

The ratio of a motor's torque capability to its volume or mass gives values for torque density and specific torque, respectively. Similarly, the motor's power capability to volume or mass gives the power density and the specific power, respectively.

These per volume or per mass quantities have some appeal because they are analogous to quantities in other kinds of selection problems. Materials, for example, might be characterized by specific tensile strength and specific modulus. Similarly, fuels can be characterized by energy density and specific energy. Diesel, for example, has a higher energy density than gasoline, but about the same specific energy. These properties are usefully scalable – twice the volume of fuel is twice the energy, regardless of the size or shape of the container.

For motors, the shape of the container makes a big difference. Motor torque is proportional to the integrated differential area elements of the gap, and the radius at which those elements are disposed with respect to the axis of rotation. In a radial flux machine, increasing the length of the gap by fifty percent increases the area while keeping the radius the same. The torque capacity should increase accordingly. In an axial flux machine, increasing the radius increases the integrated area *and* the radius at which those area elements are disposed. The torque increases nonlinearly.

Power is torque multiplied by speed. This results in even more complexity in terms of comparing motors by power density or specific power. The factors limiting speed may include losses, mechanical considerations,

balance, and ultimately strength of materials. If the rotor can be advantageously wrapped with carbon fiber to increase its chip speed, and thus achieve a higher power density, that's not really a fundamental property of the motor technology. Machines with the highest power densities tend to run at high speed, which mitigates in favor of radial flux machines with high L/D ratios if strength of materials is the limiting factor. High power density machines often require extraordinary thermal conditions to avoid failure. For example, motors for aviation that achieve impressive power densities often require extreme air flow, i.e., they do not achieve these power densities under the conditions that general purpose industrial or robot motors would have to endure. If that is the case, should the mass of the propeller be added to the calculation for specific power?

In most cases, the difficulty with torque/power densities is the parts included in the calculation. A frameless actuator motor, for example, lacks shaft, bearings, a housing, and a reduction. It *of course* has a higher torque density in the frameless state than if these necessary parts are included in the calculation.

We suggest a simplified approach. Given two motors that meet the torque and speed requirements and are integrated to a functional degree: i) do they fit in the available space? ii) with which motor does the *system* weigh less?

With respect to volume, shape is usually more important. An ECM PCB motor is essentially a flat disk. It may have more or less volume than an equivalent radial flux machine, and still not fit. In other cases, an ECM will weigh less and take up less room simply because it is designed to the exact need.

3 Example: design of an ECM quadcopter drone lift motor

3.1 Selection approach

What on the datasheet tells us that one motor will perform better than another in drone lift?

The answer, for many, would be to compare candidate motors on the basis of their *specific power*. This makes some sense, as the job of the prop is to convert power to thrust, and the less the motor weighs the better, right? Specific power is the ratio of shaft power to motor weight. Shaft power is the product of speed and torque, i.e., maximum *specific power* is a locus of torque-speed points at the edge of the safe operating area of the motor. The prop will provide a *curve* in torque-speed space, a curve that does not necessarily intersect with this locus of maximum specific power points.

None of this ensures that the motor or prop operate at their most efficient points.

When a prop is selected, the resulting system will provide varying levels of thrust as a function of rotational speed. Connected to a drone for lift, the speeds of the props will be controlled to determine ascent, attitude, etc. There is not one number, but a multitude of factors that will determine the actual delivered power and thrust, and therefore the specific power realized by the motor in operation.

The engineer is left with the unsettling feeling that selecting motors based on maximum specific power, in the absence of other information, is not the right approach. At best, comparisons of motor or even motor + subsystem numbers – whether efficiency, specific torque, specific thrust – is useful for comparisons between motors of the same type, tested in the same manner, from the same manufacturer.

3.2 Optimization Case Study

ECM's optimal motor design approach provides a different avenue for design. In part, this is based on our experience in using our motor optimizer tools, the equivalent of which is not available from other motor com-

panies. It is very easy, for example, to maximize a motor's K_m , K_t , or specific torque values. When this is done in the absence of constraints, the results are often quite obviously poor motor designs. This tends to stimulate the development of what the actual, meaningful criteria are for the desired performance. We will show here the kinds of results that this produces.

In the case of a drone motor, we might reflect on the performance of the system (including the motor), rather than the motor itself. One metric would be how long the drone can stay aloft, in a hover for example, given a particular prop, weight (exclusive of motors), and other relevant characteristics. We propose an equation for this flight time T_f , which is

$$T_f = \frac{U_b \eta_p \eta_m \eta_d}{g(M + nm)}. \quad (8)$$

The symbols in this equation are summarized below

U_b	battery energy	W s
η_p	prop efficiency	NW ⁻¹
η_m	motor efficiency	-
η_d	drive efficiency	-
g	acceleration of gravity	m s ⁻²
M	drone mass excluding motors	kg
m	motor mass	kg
n	number of motors	-

A quick dimensional analysis shows that the expression for flight time is, indeed, in seconds. We assume that U_b , M , n , g are given as part of the specification of the drone under consideration. Additionally, we assume that a prop has been chosen, so that the prop efficiency η_p is known in functional form, for example, as a function of rotational speed. The values of η_m , η_d , and m vary as a function of the motor design and the operating point, and are thus brought into the simulation for the purpose of maximizing T_f by varying the motor design.

Constraints may also be incorporated in the optimization. For example, the optimization may be done at equilibrium prop thrust, i.e., the hover condition. The voltage and current of the motor may be constrained to accommodate available voltage (batteries) and the current limitations of the controller. The optimization criteria might be evaluated over multiple flight conditions. In any case, **the result is a motor design that optimizes the quantity of interest, under the conditions of interest**, subject to the prevailing constraints.

Note that specific power, with units of W kg⁻¹ or m² s⁻³, does not appear in the quantity to be optimized.

To investigate the possibilities of designing an optimal motor according to flight time, we propose a hypothetical drone design with a system mass, M , of 5 kg, a battery energy, U_b , of 3.24 MW s corresponding to 3 kg of 300 W h kg⁻¹ batteries, and 27.5 inch KDE prop. This is a relatively large, slow prop selected for long flight time. The procedure could be repeated with other props.

Since the ECM motor is generally a flat disk and tends to grow radially, the thrust was de-rated according to the swept area of the prop and the area of the motor. This is conservative, as we found that the resulting motor did not exceed the root area of the prop significantly.

It is interesting to compare our solution to a commercial, off-the-shelf drone motor, the KDE7215. We believe this motor to be a good choice for the application.

Note that the flight time in the table above is predicted, based on the modeled results. It happens that the ECM motor above *does* have a higher specific torque than the KDE, even though optimized to a criterion that

	KDE7215	ECM
Mass [g]	555	376
Efficiency	0.77	0.78
T_f [min]	93	106

does not contain specific torque.

4 Conclusion

In this note, we review several quantities related to motor selection and performance, including differences in assumptions and meaning that can occur between manufacturers. Similarly, we argue that quantities that seem like they should be scalable within a technology, such as specific torque, may not in fact scale.

ECM suggests that developing an optimization quantity (or merit function) and set of constraints that truly address the problem at hand is more likely to provide good results than motor-only figures of merit. This criterion can be applied to catalog motors of different manufacturers for equitable comparisons, although developing easily comparable models, particularly close to the limits of the motors, may be challenging. The optimization quantity can also be validated in simulation of the whole system, or checked against existing data.

The drone lift motor that ECM designed according to the flight-time criterion shows some potential. In calculating the expected flight time of the KDE alternative, we illustrate the possibility of designing a merit criterion and then applying it to different motors for purposes of comparison. In this case, the comparison is to the ECM motor that optimizes that criterion.

It is possible – and unknown to us at this time – that a motor designed according to the general scheme of the KDE7215 but optimized in the same fashion as the ECM motor might offer better performance. However, that is a motor that doesn't exist. The ECM motor can be readily made and is in prototype at this time.

References

[1] *Comparing Peak Torque: ECM and a Typical Robot Actuator Motor*. E-Circuit Motors, Inc., 2026.